

# A stable hierarchy from Casimir forces and the holographic interpretation

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## Abstract

We show that in the Randall-Sundrum model the Casimir energy of bulk gauge fields (or any of their supersymmetric relatives) has contributions that depend logarithmically on the radion. These contributions satisfactorily stabilize the radion, generating a large hierarchy of scales without fine-tuning. The logarithmic behaviour can be understood, in a 4D holographic description, as the running of gauge couplings with the infrared cut-off scale.

# 1 Introduction

A very interesting approach to the hierarchy problem, which has inspired a great deal of research in the past few years, is the Randall-Sundrum (RS) model [1]. This consists of a 3-brane of positive tension (Planck brane) separated from a negative tension 3-brane (TeV brane) by a slice of 5D Anti-de-Sitter ( $\text{AdS}_5$ ) space. All mass scales in the theory are comparable to some Planckian cut-off scale  $M_P$ , but energy scales on the negative tension brane are redshifted (relative to those on the positive tension brane) by a geometric “warp factor”  $a = e^{-k\pi R}$ . Here  $k \lesssim M_P$  is the inverse of the AdS curvature radius and  $R$  is the thickness of the slice. By taking  $R \approx 11k^{-1}$ , the mass scales on the negative tension brane will be perceived as being only of the order  $aM_P \sim \text{TeV}$ , even if intrinsically they are comparable to the cut-off scale. A central role in this model is played by the interbrane distance  $R$ , since its value determines the hierarchy. The corresponding light degree of freedom (canonically normalized) is known as the “radion” field  $\phi$  [2]

$$\phi = fa = fe^{-k\pi R}. \quad (1)$$

Here  $f = \sqrt{6}M_P$ , where  $M_P \simeq 2.4 \times 10^{18} \text{ GeV}$ . Clearly, a complete solution to the hierarchy problem in the context of the RS model requires a stabilization mechanism for the radion. Such mechanism should explain the extremely small value of the warp factor  $a \sim \text{TeV}/M_P \sim 10^{-15}$  without the need of introducing any small parameters. Furthermore the mechanism should be stable under radiative corrections.

Soon after the RS model was proposed, Goldberger and Wise [2] showed that the radion could be successfully stabilized by introducing a classical interaction between the branes. For this purpose, they considered a bulk scalar field  $\Phi$  with mass somewhat below the cut-off scale, and with potential terms on both branes such that the vacuum expectation value of  $\Phi$  would be different on both ends of the slice. The mechanism does not involve any fine-tuning or exceedingly small parameters, and it gives the radion a mass somewhat below the TeV.

Instead of introducing an ad-hoc classical interaction between the branes, one may ask whether the Casimir energy of bulk fields may be sufficient to stabilize the radion. This issue was considered in Ref. [3], both for bulk conformal fields and for the graviton field. In both cases, however, it was found that the Casimir force would not stabilize the radion to the right value unless a tuning of parameters was invoked. Even in this case, the induced radion mass was too small.

Fortunately, this conclusion is not general. In this paper, we will show that there are certain bulk fields whose contribution to the Casimir energy density  $V(a)$  depends logarithmically on  $a$ . In particular, gauge fields (and their supersymmetric relatives) induce terms of the form  $V(a) \sim a^4/\ln a$ , which can stabilize  $a$  at very small values. This is reminiscent of the Coleman-Weinberg potential, where a small vacuum expectation value can be obtained without the need

of introducing any large or small parameters. The interesting form of  $V(a)$  can be understood by using the AdS/CFT correspondence. The RS model with gauge bosons in the bulk corresponds to a 4D conformal field theory (CFT) where a certain global symmetry has been gauged. We will see that the logarithmic contribution to  $V(a)$  arises from the running of the gauge coupling.

The plan of the paper is the following. In Section 2, the bulk fields giving logarithmic contributions to  $V(a)$  will be identified by using the AdS/CFT correspondence. In Section 3, the radion effective potential for these specific bulk fields is computed in the slice of  $\text{AdS}_5$ . The last Section is for conclusions.

## 2 The radion potential from CFT

Here we will consider the 4D holographic version of the RS model (a slice of  $\text{AdS}_5$ ) to argue that there are sizable logarithmic contributions to the radion potential.

The AdS/CFT correspondence relates a 5D theory in AdS to a 4D strongly coupled CFT [5] with large number of “colors”  $N$ . This is based on the identification of the 5D fields at the AdS boundary,  $\Phi(x)$ , as the sources of the CFT operators

$$\mathcal{L} = \Phi(x)\mathcal{O}(x). \quad (2)$$

The correspondence says [6] that, for bosons, the masses  $M$  of the 5D fields are related to the dimension of the CFT operators according to

$$\text{Dim}[\mathcal{O}] = \alpha + 2, \quad \text{with} \quad \alpha = \sqrt{\left(\frac{s}{2}\right)^2 + \frac{M^2}{k^2}}, \quad (3)$$

where  $s = (4, 2, 4)$  for scalars, spin-1 fields and the graviton respectively. For fermions [7], the 5D fields  $\Psi$  at the AdS boundary must be subject to a projection  $\Psi_{\pm} = \frac{1}{2}(1 \pm \gamma_5)\Psi$  for  $M \geq 0$  and  $M < 0$  respectively. Only half of the degrees of freedom of the fermions are identified as sources,  $\mathcal{L} = \bar{\Psi}_{\pm}(x)\mathcal{O}_{\pm}(x) + h.c.$ , where

$$\text{Dim}[\mathcal{O}_{\pm}] = \alpha_{\pm} + \frac{3}{2}, \quad \text{with} \quad \alpha_{\pm} = \left| \frac{M}{k} \pm \frac{1}{2} \right|, \quad (4)$$

for  $M \geq 0$  and  $M < 0$  respectively.

This correspondence can also be extended to the case of a slice of  $\text{AdS}_5$ . The Planck brane corresponds to an ultraviolet cutoff at  $p = k \sim M_P$  in the 4D CFT [8], while the TeV brane corresponds to an infrared cutoff at  $p = ak \sim \text{TeV}$  [9, 10]. Therefore a slice of  $\text{AdS}_5$  corresponds to a slice of CFT in the momentum-scale. The ultraviolet cutoff has two important

consequences. First, higher dimensional terms, suppressed by the cutoff  $k$ , can appear in the theory. Second, the CFT can generate, via Eq. (2), kinetic terms for the sources  $\Phi(x)$ , which become dynamical. For the case where  $\mathcal{O}$  is a conserved current, this effect corresponds to a gauging of the associated conserved symmetry in the CFT. For example, gravity and gauge theories in  $\text{AdS}_5$  are dual to a CFT coupled to gravity and to gauge bosons in 4D [8, 9].

Since the conformal symmetry is spontaneously broken in the infrared [9, 10], a mass gap proportional to  $ka$  is generated. Bound states of CFT are expected to form, analogous to the mesons and baryons in QCD. Neglecting the interaction with the sources, the bound state masses scale linearly with  $a$ :

$$m_{\text{CFT}} \simeq ka. \quad (5)$$

However, once the interaction Eq. (2) is taken into account, the CFT bound states mix with the sources  $\Phi$ . This is analogous to the mixing of the  $\rho$  meson with the photon in QCD. The new mass eigenvalues, which will now have a nontrivial dependence on  $a$ , are identified with the Kaluza-Klein (KK) masses in the slice of  $\text{AdS}_5$  [9]. In particular, we shall be interested in the case where the correction to the mass behaves logarithmically.

Let us consider first the case of an abelian gauge theory in  $\text{AdS}_5$ . In the CFT holographic picture, this corresponds to adding the perturbation  $\mathcal{L} = gA_\mu(x)J^\mu(x)$ , where  $\text{Dim}[J]=3$ . The CFT gives a contribution to  $\langle JJ \rangle$  dictated by the conformal symmetry

$$\langle JJ \rangle = cNp^2 \ln p, \quad (6)$$

where  $c$  is a  $p$ -independent constant. The CFT correlator Eq. (6) renormalizes the photon propagator. Like in QED, this renormalization can be incorporated in the running of the gauge coupling:

$$g^2(\mu) = \frac{g^2(k)}{1 + cNg^2(k) \ln \frac{k}{\mu}} \sim \frac{1}{cN \ln \frac{k}{\mu}}, \quad (7)$$

where in the last approximation we have taken the large  $N$  limit. As mentioned above, the photon mixes with the spin-1 CFT bound-states  $V$ . Assuming that  $g$  is small, we can treat this mixing as a perturbation, and estimate the  $\mathcal{O}(g^2)$  correction to the mass of  $V$  (given by Eq. (5)). This correction arises from the diagram of Fig. 1, which gives

$$\delta m_V^2 \sim g^2(\mu \sim m_V) f_V^2 m_V^2, \quad (8)$$

where  $f_V \propto \langle 0|J|V \rangle$ . In a large  $N$ , we have [12]  $f_V \sim \sqrt{N}$  and we obtain for small  $a$

$$\delta m_V^2 \sim \frac{k^2 a^2}{\ln a}. \quad (9)$$

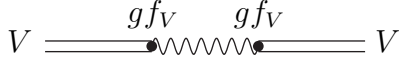


Figure 1: Correction to the mass of  $V$ .

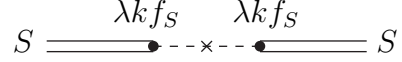


Figure 2: Correction to the mass of  $S$ .

We see that the mass of  $V$  has acquired a logarithmic dependence on the warp factor  $a$ . Therefore its contribution to the vacuum energy will be of the order

$$V(a) \sim m_V^4 \ln \frac{m_V}{\mu} \sim k^4 a^4 \left(1 + \frac{c_1}{\ln a}\right), \quad (10)$$

where  $m_V^2 \simeq m_{\text{CFT}}^2 + \delta m_V^2$ , and we have used  $\mu \sim m_V$ . Going back to the 5D dual theory, we conclude that a gauge theory in a slice of  $\text{AdS}_5$  must generate at the one-loop level a radion potential with terms of order  $a^4/\ln a$ .

Similar arguments can be applied also to massless scalars  $\phi$  coupled to CFT operators,  $\mathcal{L} = \lambda\phi(x)\mathcal{O}(x)$ . If  $\text{Dim}[\mathcal{O}]=3$ , then  $\lambda$  is dimensionless and is renormalized by the CFT in the same way as the gauge coupling  $g$ . We have then the same situation as the one of the photon described above and we expect that terms of order  $a^4/\ln a$  will be induced in the radion potential. In the 5D dual picture, we find from Eq. (3) that a scalar operator of  $\text{Dim}[\mathcal{O}]=3$  is associated with a bulk scalar with mass  $M^2 = -3k^2$ . The massless scalar  $\phi$  must arise from the 5D scalar field after dimensional reduction. In order to have a massless mode in the spectrum, the scalar field must have a boundary mass term equal to [11]  $[2k^2\delta(y) - 2k^2\delta(y - \pi R)]\phi^2$ . We must notice, however, that this massless scalar is not stable under radiative corrections, unless supersymmetry is invoked. This problem does not arise for the fermions. We can have a massless chiral fermion  $\Psi_+$  coupled to a CFT operator  $\mathcal{L} = \lambda\bar{\Psi}_+(x)\mathcal{O}_+(x)$  with  $\text{Dim}[\mathcal{O}_+]=5/2$ . Again  $\lambda$  is dimensionless and logarithmically dependent on the momentum, guaranteeing that the vacuum energy has terms proportional to  $a^4/\ln a$ . From Eq. (4), this corresponds to a bulk fermion with mass  $M = k/2$  in  $\text{AdS}_5$ .

It should be noted that the three cases presented above are related by supersymmetry [11]. In  $\text{AdS}_5$  the photon is in a vector supermultiplet that contains a fermion of mass  $M = k/2$ . On the other hand, a fermion of mass  $M = k/2$  can also be contained in a hypermultiplet with a scalar of bulk mass  $M^2 = -3k^2$  and boundary mass term  $[2k^2\delta(y) - 2k^2\delta(y - \pi R)]\phi^2$ . This scalar, fermion and the photon correspond to the three cases found above. All of them have  $\alpha = 1$  [see Eq. (3)]. In the vector multiplet, there is also a scalar of mass  $M^2 = -4k^2$  and Dirichlet boundary conditions (which therefore has no 4D massless mode). This has  $\alpha = 0$ . By supersymmetry this case must also induce a  $V(a)$  with terms  $\sim a^4/\ln a$ . To analyze this last case, let us go back to the CFT holographic picture.

Let us consider a scalar,  $\phi$ , with mass of order  $k$ , coupled to a CFT operator of  $\text{Dim}[\mathcal{O}]=2$ :  $\mathcal{L} = \lambda k\phi(x)\mathcal{O}(x)$ . Since the coupling is of order  $k$ , an unsuppressed correction to the mass of

a scalar CFT bound-state  $S$  can be generated from the diagram of Fig. 2. The correction is of order  $\delta m_S^2 \sim \lambda^2 f_S^2 m_S^2$  where  $f_S \propto \langle 0 | \mathcal{O} | S \rangle$ . The conformal symmetry guarantees that  $\lambda$  evolves logarithmically with the momentum. Therefore  $\delta m_S^2$  and one-loop vacuum energy induced by this state have a logarithmic dependence on  $a$ . Contrary to the above cases, in which we needed dynamical sources to generate the diagram of Fig. 1, here the scalar  $\phi$  can be considered an auxiliary (nondynamical) field <sup>1</sup>. In the AdS picture, this effect will come from a bulk scalar of mass  $M^2 = -4k^2$  with boundary conditions such that the dimensionally reduced 4D theory has no massless modes, e.g., Dirichlet boundary conditions.

### 3 Radion effective potential in RS

As mentioned in the introduction, the radion develops an effective potential at the one-loop level due to the Casimir energy of bulk fields. This effect has been studied in Ref. [3], for the case of bulk conformal fields and the graviton. More general bulk fields were considered in Refs. [4]. However, it appears that the interesting behavior of the specific bulk fields mentioned in the previous section has been overlooked so far.

The starting point for the calculation of the effective potential is the spectrum of KK states of the different fields, as a function of the radion expectation value  $a$ . This spectrum depends on the spin, as well as on the boundary conditions on the branes. In general, it is given by the roots of the following equation [1, 11]

$$F(m_n) = J_\alpha^T(m_n/ak)Y_\alpha^P(m_n/k) - J_\alpha^P(m_n/k)Y_\alpha^T(m_n/ak) = 0. \quad (11)$$

The superindices  $T$  and  $P$  stand for TeV and Planck branes respectively. For fields with Neumann boundary conditions, the functions  $J_\alpha^i$  are defined as

$$J_\alpha^i(z) = \epsilon_i J_\alpha(z) + z J_{\alpha-1}(z), \quad (i = T, P) \quad (12)$$

where  $J_\alpha$  are the usual Bessel functions and

$$\epsilon_i \equiv s2 - \alpha - r_i, \quad \text{for bosons}, \quad (13)$$

$$\epsilon_i \equiv s2 - \alpha_\pm \pm \frac{M}{k}, \quad \text{for } \Psi_\pm \text{ fermions}. \quad (14)$$

The parameters  $\alpha$  and  $s$  are given in Eqs. (3) and (4). The functions  $Y_\alpha^i$  are defined by a similar equation, with  $Y_\alpha$  replacing  $J_\alpha$ . The parameters  $r_T$  and  $r_P$  represent the mass terms on the TeV and on the Planck brane respectively. For instance, the mass term for a boson  $\Phi$  including the

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<sup>1</sup>The kinetic term of  $\phi$  does not play any role in the correction to the mass of  $S$ . The effect of  $\phi$  is equivalent to inserting a dimension 4 operator  $\mathcal{L} = \lambda^2(\mu)\mathcal{O}\mathcal{O}$  at the cutoff scale.

brane contributions takes the form  $[M^2 + 2kr_P\delta(y) - 2kr_T\delta(y - \pi R)]\Phi^2$ . In the case of gravitons ( $\alpha = 2$ ) and massless vectors ( $\alpha = 1$ ) we have [3, 11]

$$\epsilon_i = 0, \quad (15)$$

and therefore  $J_\alpha^i(z) = zJ_{\alpha-1}(z)$ . The same applies to the corresponding fermionic superpartners [11].

The case with Dirichlet boundary conditions can be obtained from the previous expressions by taking the limit

$$\epsilon_P \rightarrow -\infty, \quad \epsilon_T \rightarrow \infty. \quad (16)$$

For bosons this corresponds to infinite masses on the brane. In the limit Eq. (16) the second term in Eq. (12) becomes negligible and we have  $J_\alpha^i(z) = J_\alpha(z)$ , and  $Y_\alpha^i(z) = Y_\alpha(z)$ , where we have dropped an irrelevant multiplicative constant.

From Eq. (11) we can get the masses for the KK states of the photon. As we said, they correspond to the CFT bound-states in the mass eigenstate basis. In the limit  $a \ll 1$  we obtain

$$m_n \simeq \left(n - \frac{1}{4}\right) \pi k a - \frac{1}{\ln a} \frac{\pi k a}{2}, \quad n = 1, 2, \dots, \quad (17)$$

that shows the  $a$ -dependence expected from the CFT arguments given in Section 2.

The sum over KK contributions to the effective potential for the radion can be expressed as a contour integral in the complex  $m_n$  plane, involving the logarithmic derivative of the function  $F(m_n)$ . After some manipulations, this integral can be written as [3, 4]

$$V(a) = k^4 16\pi^2 [A + a^4 B] + \Delta V(a), \quad (18)$$

where

$$\Delta V(a) = (-1)^S g_* k^4 a^4 16\pi^2 \int_0^\infty dt \, t^3 \ln |1 - K_\alpha^T(t) I_\alpha^P(at) I_\alpha^T(t) K_\alpha^P(at)|. \quad (19)$$

Here,  $g_*$  is the number of physical polarizations, and  $S = 0, 1$  for bosonic and fermionic degrees of freedom respectively. The first two terms in Eq. (18) correspond to renormalizations of the Planck and TeV brane tensions, respectively. The piece proportional to  $A$  is just a constant term which must be adjusted to give a zero cosmological constant. The functions in the integrand in Eq. (19) are given in terms of the Bessel functions  $I_\alpha$  and  $K_\alpha$  through the following relations:

$$I_\alpha^i(\rho) = \epsilon_i I_\alpha(\rho) + \rho I_{\alpha-1}(\rho), \quad (20)$$

$$K_\alpha^i(\rho) = \epsilon_i K_\alpha(\rho) - \rho K_{\alpha-1}(\rho). \quad (21)$$

Let us now find the form of  $\Delta V(a)$  in the limit of a large hierarchy  $a \ll 1$ . The first thing to notice is that  $K_\alpha^T(t)/I_\alpha^T(t) \sim e^{-2t}$  for  $t \gg 1$ , and therefore, only the range  $t \sim 1$  will make a significant contribution to the integral in Eq. (19). The dependence on the radion  $a$  of this integral arises from  $K_\alpha^P(at)/I_\alpha^P(at)$ , which can be expanded for small  $at$  as

$$K_\alpha^P(at)I_\alpha^P(at) = (\Gamma(\alpha)\Gamma(1+\alpha)(\epsilon_P + 2\alpha)(at2)^{-2\alpha} \{ \epsilon_P 2 + (at2)^2 11 - \alpha \} - \pi 2 \sin \alpha \pi) [1 + O(a^2 t^2)] . \quad (22)$$

Since  $a$  is extremely small, and  $\alpha > 0$ , the leading term is proportional to  $\epsilon_P$ . Hence, the behavior will be very different for  $\epsilon_P = 0$  or for  $\epsilon_P \neq 0$ .

Of particular interest will be the case of a massless gauge boson ( $\epsilon_P = 0$ ,  $\alpha = 1$ ), and a scalar with  $M^2 = -4k^2$  ( $\epsilon_P \neq 0$ ,  $\alpha = 0$ ), where the right hand side of Eq. (22) behaves <sup>2</sup> as  $\ln a$ . This will imply a correction to the effective potential which behaves as  $1/\ln a$ , as suggested by the AdS/CFT arguments presented in the previous section. These two special cases will be dealt with in the following subsections.

When  $\alpha$  is not close to 0 or 1, one can show from Eq. (22) that  $\Delta V(a)$  behaves either as  $a^4$  or as a higher power of  $a$ . Terms proportional to  $a^4$  can be absorbed in a redefinition of  $B$ , whereas the higher powers have a negligible effect. Hence, we shall disregard these cases in the following discussion.

### 3.1 Gauge fields ( $\alpha = 1$ )

As mentioned above, for a massless gauge field we have  $\epsilon_P = 0$  and  $\alpha = 1$ . In the limit  $|\alpha - 1| \ll |1/\ln a|$ , Eq. (22) reduces to

$$K_\alpha^P(at)I_\alpha^P(at) \approx \ln(at/2) + \gamma + O(a^2 t^2), \quad (23)$$

where  $\gamma$  is Euler's constant. Since we are interested in the situation where  $|\ln a| \gg 1$ , the argument of the logarithm in Eq. (19) is close to unity and we have

$$V(a) \approx k^4 a^4 16\pi^2 (B - g_* (-1)^S \beta_1(\epsilon_T) \ln a) + const., \quad (24)$$

where we have introduced

$$\beta_\alpha(\epsilon_T) \equiv \int_0^\infty dt t^3 K_\alpha^T(t) I_\alpha^T(t). \quad (25)$$

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<sup>2</sup>This can be seen from the expansion  $a^{2(1-\alpha)} \approx 1 + 2(1-\alpha) \ln a$  and  $a^{-2\alpha} \approx 1 - 2\alpha \ln a$  in the limit  $\alpha \rightarrow 1$  and  $\alpha \rightarrow 0$  respectively.



The extremum of the potential is determined by the condition

$$\ln a \approx (-1)^S g_* \beta_1(\epsilon_T) / B, \quad (26)$$

that corresponds to a minimum if the radion mass

$$m_\phi^2 \approx (-1)^S g_* \beta_1(\epsilon_T) (2\pi \ln a)^2 a^2 k^4 f^2, \quad (27)$$

is positive. Eq. (26) shows that a exponentially small  $a$  can be obtained from parameters of order one. In particular the large  $M_P/\text{TeV}$  value can be obtained if the r.h.s. of Eq. (26) takes the value  $\approx -35$ . For massless gauge bosons, ( $\epsilon_T = 0$ ,  $S = 0$ ) we have  $\beta_1(0) \approx -1.005 < 0$  and the radion mass, Eq. (27), becomes negative. Nevertheless, the corresponding fermionic field (with a mass  $M = k/2$ ) can stabilize the radion, giving it a mass  $m_\phi \approx g_*^{1/2} a k^2 / (2\pi f \ln a) \sim 4 \times 10^{-3} g_*^{1/2} (a k^2 / f)$ . We may also consider the situation with different boundary conditions on the branes. If we use Dirichlet conditions on the Planck brane, then it is immediate from Eq. (22) with  $\epsilon_P \rightarrow -\infty$  that  $\Delta V \propto a^6$ . This is a negligibly small correction. If we keep the Neumann boundary condition on the Planck brane ( $\epsilon_P = 0$ ) and impose Dirichlet conditions on the TeV brane ( $\epsilon_T \rightarrow \infty$ ), we have  $\beta_1(\infty) \approx 2.33$ . In this case, it is the boson (and not the fermion) that would stabilize the radion. In the 4D dual this corresponds to break the gauge symmetry by giving to the gauge boson a mass of order TeV.

### 3.2 Fields with $\alpha \approx 0$

From Eq. (3), a scalar field with  $\alpha = 0$  has a bulk mass  $M^2 = -4k^2$ . If we add mass terms on the branes,  $r_P = r_T = 2$  (so that  $\epsilon_T = \epsilon_P = 0$ ), then it is easy to show from Eq. (11) that there is a zero mode in the spectrum, and all other mass eigenstates are positive. In general, we will not have tachyons in the spectrum of KK masses provided that

$$\epsilon_T \epsilon_P \ln a + \epsilon_T - \epsilon_P \geq 0, \quad (28)$$

$$\epsilon_T \geq \epsilon_P. \quad (29)$$

These two conditions are automatically satisfied if  $\epsilon_T \geq 0$  and  $\epsilon_P \leq 0$ , which we now assume. In the case where  $\epsilon_P \neq 0$  and  $\alpha$  is small, we have from Eq. (22):

$$K_\alpha^P(at) I_\alpha^P(at) \approx - [\ln(at/2) + \gamma + (1/\epsilon_P) + O(a^2 t^2)]. \quad (30)$$

Provided that  $|\ln a + 1/\epsilon_P| \gg 1$ , the integral in Eq. (19) can be approximated as

$$V(a) \approx k^4 a^4 16\pi^2 (B + g_* (-1)^S \beta_0(\epsilon_T) \ln a + 1/\epsilon_P) + \text{const.}, \quad (31)$$

where  $\beta_0(\epsilon_T)$  is given in Eq. (25). The extremum of the potential is determined by the condition

$$x \equiv \ln a + (1/\epsilon_P) \approx (-1)^{S+1} g_* \beta_0(\epsilon_T) / B, \quad (32)$$

and the radion mass at the extremum is given by

$$m_\phi^2 \approx (-1)^{S+1} g_* \beta_0(\epsilon_T) (2\pi x)^2 a^2 k^4 f^2. \quad (33)$$

A boson ( $S=0$ ) with small  $\epsilon_T$  can lead to a positive radion mass since  $\beta_0(\epsilon_T \ll 1) \approx -\beta_1(\infty) \approx -2.33$ . The mass will be largest for the smaller possible value of  $|x|$ . Given that  $\ln a \approx -35$ , and since here we are considering negative  $\epsilon_P$ , the best we can do is to consider Dirichlet boundary conditions on the Planck brane  $\epsilon_P \rightarrow -\infty$ , which corresponds to  $|x| \approx 35$ . The mass of the radion is then comparable to the one obtained in Eq. (27).

We may consider more general situations, where for instance  $\epsilon_P > 0$ , provided that we stay in the range of values of  $a$  where the conditions of Eqs. (28) and (29) are satisfied. In the general case, the approximation in Eq. (31) will not be valid. However, it will still be true that  $\Delta V/a^4$  depends on  $a$  through the combination  $x$  defined in Eq. (32). For  $\epsilon_P > 0$ , the conditions Eqs. (28) and (29) require  $x > 0$ , but in contrast with the previous case, there is no requirement that  $x$  should be large. Hence, one may expect the existence of minima of  $V(a)$  at  $x \sim 1$ , where the mass of the radion is of order TeV without the need of fine adjustments in the parameters. Since the analytic approximation in Eq. (31) breaks down, one has to proceed numerically. For instance, we find that for  $\epsilon_T = 0.5$  and  $\epsilon_P \approx 1/39$ , the potential has a minimum for  $B \approx 0.3g_*$  where  $x \sim 1$  and where the mass of the radion is given by  $m_\phi \approx 10^{-2} g_*^{1/2} (ak^2/f)$ . This is slightly larger than the mass in Eq. (27), but not dramatically so. Also, in this case one has to worry about the neighboring regime of smaller values of  $a$ , where Eq. (28) is violated. This could make the minima with  $x \sim 1$  metastable, which makes the case with  $\epsilon_P > 0$  somewhat less appealing.

## 4 Conclusions

We have shown that in the RS model the radion can be stabilized by the Casimir energy of certain bulk fields, leading to the observed hierarchy  $a \sim \text{TeV}/M_P$  without the need of introducing small parameters. The main ingredient is the appearance of Casimir energy contributions that depend logarithmically on the radion  $a$ . These contributions arise from gauge fields in the bulk or from any of their supersymmetric relatives: a fermion with mass  $M = k/2$  or a scalar with mass  $M^2 = -4k^2$ . This effect can be understood by using AdS/CFT and relating the logarithmic terms in  $V(a)$  with the running of the gauge coupling in the 4D dual theory.

Hence the Casimir force provides a natural alternative to the Goldberger-Wise mechanism for stabilizing the radion in the RS model, as required for a complete solution to the hierarchy problem.

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